MULTI(SCALE)GRAVITY: A TELESCOPE FOR THE MICRO-WORLD

Ian I. Kogan
Department of Physics, Theoretical Physics, 1 Keble Road,
Oxford OX1 3NP, England

This is a talk presented at XXXVIth Rencontres de Moriond, ElectroWeak Interactions and Unified Theories, March 2001. A short review of modern status of multigravity, i.e. modification of gravity at both short and large distances is given.

1 Introduction

The Brane Universe scenario is actually quite an old idea [1,2,3,4,5,6]. [a] Recently it has been subject of renewed interest [11,12,13] with the realization that such objects are common in string theory. In particular, there has been a lot of activity on warped brane constructions in five spacetime dimensions, motivated by heterotic M-theory [14,15,16] and its five dimensional reduction [17,18]. In the context of these constructions, one can localize gravity on the brane world having four dimensional gravity even with an extra dimension of infinite extent [19,20], or can generate an exponential mass hierarchy on a compact two brane model as it was done in the Randall - Sundrum (RS1) model [21], providing a novel geometrical resolution of the Planck hierarchy problem. For more details on warped models see an excellent review of [22]. Usually embedding of Standard Model and General Relativity into any multidimensional construction gives rise to all possible sorts of new effects in a Micro-world, i.e. we are definitely going to modify a

[a] The author learnt about Regge and Teitelboim (RT) paper [1] which is virtually unknown in a modern "Brane community" from [7]. For more information about RT see [8] and references therein; see also [9] for critical comparison between RT approach and four-dimensional general relativity. In [6] there are also references on even earlier discussions on what is called now "Brane Universe" - but was called "Embedding Problem" in 1965. Proceedings of seminar on embedding problems edited by I.Robinson and Y.Ne'eman are published in [10]. It seems that the more we work on Brane Worlds the more ideas from the past become visible (the same way as light from distant stars we could not see earlier). Perhaps when we reach some future frontier an old picture about Earth (read "Universe") standing on three elephants standing on one big turtle will make some sense ?

short-distance physics. The subject of this talk is to show that sometimes one can get a very drastic modification of the laws of gravity at ultra-large, cosmological scale.

2 Modification of gravity at large distances

The force of gravity between two bodies with masses m_1 and m_2 depends on a distance r between them as

$$F = G\frac{m_1 m_2}{r^2} \tag{1}$$

This law was used by Newton to explain the famous Kepler laws[b].

Why it is inverse square law ? The modern answer is the following - because we live in a three-dimensional space (four-dimensional space-time) and graviton is massless !

How well do we know this law ? We definitely do not know it below $1 - 0.1mm$ as well we do not know it at distances comparable with the size of observable Universe. Can we modify it at large scales ? [c] There are a lot of publications about possible modification of Newtonian gravity at large scales, for example famous Milgrom proposal [25,26] as well as many others - for the list of references see [27]. The problem with these modifications that it is hard to reconcile them with General Relativity.

In General Relativity the deviation from the inverse square law must be due to a very small mass of a graviton. However it seems that there is no consistent four-dimensional theory with massive graviton [28] [d]

2.1 Giant see-saw with us in the middle

Before we shall discuss multidimensional theories let us ask a question - what is the largest see-saw one can make ? Well, in physics see-saw means that there are 3 scales - large, small and the third one whis is just the geometric average of the first two. Than the largest possible see-saw is a such one that the large scale is the largest possible scale and the second one - is the smallest possible scale. The largest scale we have now is the size of observable Universe $R_U = 10^{28}$ cm and the smallest one must be the Planck length $r_P = 10^{-33}$ cm. So what will be the third, intermediate scale L ?

$$r_P\, R_U = L^2, \quad L^2 = 10^{-5} cm^2, \quad L \sim 10^{-1} - 10^{-2} mm \tag{2}$$

It is really amusing that in the middle of the largest see we can find a place for ourselves. This scale is a “biological” one - close to the size of our cells, etc...

But of of course this new scale L can be related to some non biological physics too. First of all it corresponds to mass scale $10^{-3} - 10^{-4}$ ev which is very close to the current limit on cosmological constant[e] (neutrino physics also may be related to this energy scale). The reason for this is very simple - the curvature radius R corresponding for the cosmological constant Λ is given by $R^{-2} = \Lambda r_P^2$. If vacuum energy is $1/L$ then $\Lambda \sim (1/L)^4$ and we get $R^{-1} \sim r_P/L^2$. The

---

[b] Actually there are strong evidence (see for example very interesting book [23] and references therein) that Robert Hooke suggested the inverse square law to Newton. Hooke used the third Kepler's law $R^3\omega^2 = const$ and expression for centrifugal force $\omega^2 R$ acting on a particle moving with a constant angular velocity ω along a circle of radius R to get the force proportional to $1/R^2$. It was also known to Halley and Wren. But only Newton was able to derive all three Kepler's laws. As another curious element of Oxford involvement in gravity research in the 17 century it is interesting to note that first meetings of Royal Society were held in Wadham College, Oxford

[c] In this talk we are not going to discuss the modification of gravity below $1mm$ - short-distance modifications are inevitable in any quantum/multidimensional/etc scenario. One can get it for example in a pure four-dimensional theory with R^2 terms - see [24] and references therein.

[d] For early discussion of phenomenological limits on graviton mass see [29].

[e] For recent review - see [30].

current limit for cosmological constant is such that respective curvature radius $R \sim R_U$ - so we again have Eq. 2.

Another interesting fact about the scale L is that this is just the boundary between quantum and classical behaviour for particle with a Planck mass scale. If one considers the wave packet of intimal size L for particle with mass m the wave packet size at moment t will be (we use units $c = \hbar = 1$)

$$L^2(t) = L^2 + \frac{t^2}{m^2 L^2} \tag{3}$$

from which one can see that the transition time T between classical and quantum behaviour is given by $T \sim mL^2$. For Planck mass particle $m = r_P^{-1}$ the scale $L \sim 10^{-1} - 10^{-2} mm$ corresponds to transition time equal to the age of Universe $T \sim R_U$ as we can see from Eq. 2.

As we shall see later the giant see-saw means something else - modification of Newtonian gravity at large scales can not be excluded [31]. But to see how does Eq. 2 lead to such a surprising possibility we have to discuss first how does one get four-dimensional gravity from a multi-dimensional gravity.

2.2 Four-dimensional gravity from extra dimensions

Let us start from a simple question - how one can get a four-dimensional gravity from higher dimensions. The Einstein-Hilbert action in D-dimensional space-time is

$$S = M_F^{D-2} \int d^{D-4}y d^4x \sqrt{G} G^{MN} R(G)_{MN} \tag{4}$$

where M_F is some fundamental scale and G_{MN} is a full D-dimensional metric describing in a flat space limit graviton with $D(D-3)/2$ degrees of freedom. For a four-dimensional gravity to exist we must have finite Planck mass M_P which is defined as

$$M_P^2 = M_F^{D-2} \int d^{D-4} y \sqrt{G} \tag{5}$$

This existence of a four-dimensional gravity depends on what is the volume of extra dimensions. If it is finite (even if the space is non-compact) we bound to have four-dimensional gravity. If it is infinite - massless graviton will not exist (but we may have a massive one).

But of course besides massless graviton we shall have massive excitations. For example in a canonical KK case of a space $R^4 \times S^1$ there is a a whole massive KK tower. Each massive graviton has 5 degrees of freedom - precisely the same number as massless graviton in five dimensions $5(5-3)/2 = 5$. Massless graviton has only 2, but then there is a massless vector field - graviphoton $G_{\mu 5}$ with 2 degrees of freedom and scalar G_{55} - another degree of freedom. So at each muss level we have 5 degrees of freedom as it must be - compactification can not change the total number of the degrees of freedom.

So why we can not use these massive gravitons to modify gravity ? We can - but this will be modification at *short* distances. The mass spectrum is equidistant and for a circle with radius R is given by $m_n = n/R$. We just can not take m_1 to be to light - the moment the first mode is important we start to see the whole KK tower - and we open the new dimension. The same is true for all known compact manifolds - they all have regular spectra. There is no way to modify gravity at large distances unless we have something like our giant see-saw - which one can write in another form

$$M_P m_1 = m_2^2 \tag{6}$$

where M_P is a Planck mass, m_1 is an ultra-light mass of first (or several first) excitation(s) corresponding to a cosmological scale, i.e. $m_1 \sim R_U^{-1}$, where m_2 corresponds to much higher

mass scale corresponding to much smaller length L below sub-millimeter scale. If there are no graviton masses between m_1 and m_2 we do not have any obvious violation of a Newton law between L and R_U - what we see is determined by massless and ultra-light graviton and other excitations become important only at short distances smaller than L. Of course we can take m_1 a little bit bigger and without any contradiction with experiment modify gravity at let say 1% of a Universe size [31]. We can do other things (some of which will be discussed later) - but independently on what model we are going to deal with there is one important general rule: we must have a spectrum with extremely light (ultra-light) states.

2.3 Multigravity - ultra-light states from multilocalization

We must have some natural mechanism producing these ultra-light states - and as it always happens in physics we want this mechanism to be natural. It means that in some limit our ultra-light states must become massless. But then it is clear that we have to start from some configuration which support massless graviton (and we have already discussed that there are many ways to do it) and after that do something unusual - take several configurations like that and let them "talk to each other". ¿From the basic principles of quantum mechanics we know that several degenerate levels will split - and if the system is stable and there are no tachyons, we shall get a massless ground state and all other states become ultra-light. This is true for compact spaces - noncompact spaces can be studied as limiting cases of compact ones.

So the non-trivial task is to figure out how is it possible to take several configurations supporting massless graviton and let them "talk to each other" ? The only solution known now is based on multibrane constructions. There is no place here to discuss technical details so we shall explain only general concepts, for more details reader is advised to look at original papers.

It is obvious that in order the scenario of multi-localization to be realized it is necessary to have a mechanism that induces the localization of the fields under consideration. For the case of the graviton, it has been shown that it can be localized if the geometry of the extra dimension is non trivial (with specific properties).Remember that what we need is more than simply a massless graviton (in which case finite volume of extra dimensions will be enough) - but massless graviton which is *localized*. As an illustration let's consider a non-factorizable geometry with one extra dimension. In this scenario the fifth dimension y is compactified on an orbifold, S^1/Z_2 of radius R, with $-L \leq y \leq L$. The five dimensional spacetime is a slice of AdS_5 which is described by[f]:

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{7}$$

where the warp factor $\sigma(y)$ depends on the details of the model considered. For the moment we assume that we have a model with a number of positive and negative tension flat branes (the sum of the brane tensions should be zero if one wants flat four dimensional space on the branes) and that this function is known (it can be found by looking the system gravitationally).

Starting from a five dimensional Lagrangian in order to give four dimensional interpretation to the five dimensional fields one has to go through the dimensional reduction procedure. This procedure includes the decomposition of the five dimensional fields (actually these arguments can be applied not only to gravity and this is why we do not specify the tensor structure) $\Phi(x, y)$ in KK states:

$$\Phi(x,y) = \sum_{n=0}^{\infty} \Phi^{(n)}(x) f^{(n)}(y) \tag{8}$$

where $f^{(n)}(y)$ is a complete orthonormal basis. The idea behind this KK decomposition is to find an equivalent description of the five dimensional physics associated with the field of interest,

[f] We will assume that the background metric is not modified by the presence of the bulk fermion, that is, we will neglect the back-reaction on the metric from the bulk fields.

through an infinite number of KK states with mass spectrum and couplings that encode all the information about the five dimensions. The function $f^{(n)}(y)$ describes the localization of the wave function of the n-th KK mode along the extra dimension. In order the above to be possible, it can be shown that $f^{(n)}(y)$ should obey a second order differential equation which after a convenient change of variables and/or a redefinition[g] of the wave-function, reduces to an ordinary Schrödinger equation:

$$\left\{-\frac{1}{2}\partial_z^2 + V(z)\right\} \hat{f}^{(n)}(z) = \frac{m_n^2}{2}\hat{f}^{(n)}(z) \tag{9}$$

The mass spectrum and the wave-functions (and thus the couplings) are determined by solving the above differential equation. Obviously all the information about five dimensional physics is contained in the form of the potential $V(z)$. For example in the case of the graviton the positive tension branes correspond to attractive δ-function potential wells whereas negative tension branes to δ-function barriers. The form of the potential between the branes is determined by the AdS_5 background.

2.4 Example: +–+ model as interpolation between Bigravity and quazilocalization

The first model where multigravity been found is the so-called $+-+$ or bigravity[h] model[31], which consists of two positive branes with tensions Λ located at the fixed points of a S_1/Z_2 orbifold with one negative brane with tension $-\Lambda$ which can move freely in between. The next one is a GRS model[32] which can be obtained from this model by cutting the negative brane in half, i.e. instead of one $''-''$ brane with tension $-\Lambda$ one can take two branes with negative tension $-\Lambda/2$ each and then move them apart. Generic case was suggested and studied in[33,35] and is called $+--+$ model (see Fig.1)

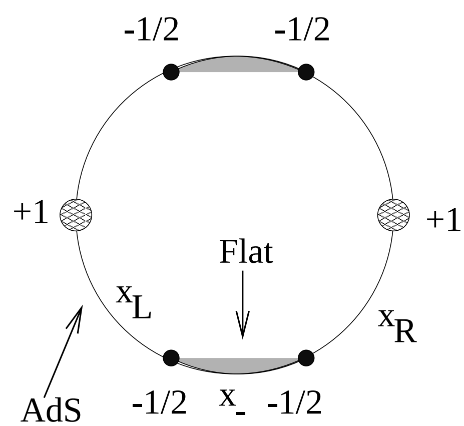

Figure 1: $+--$ model with two $''+''$ branes at the fixed points and two moving $''-1/2''$ branes. In the limiting case when $x_- \to 0$ we have a $+-+$ model. The GRS model can be obtained in the opposite limit $x_- \to \infty$ The whole configuration can be considered as two GRS models connected via flat space.

This model consists of four parallel 3-branes in an AdS_5 space with cosmological constant $\Lambda < 0$. The 5-th dimension has the geometry of an orbifold and the branes are located at $y_0 = 0$, $y_1 = l_L$, $\quad y_2 = l_L + l_{--}$ and $y_3 = y_2 + l_R$, where y_0 and y_3 are the orbifold fixed points[i]

---

[g] The form of the redefinitions depend on the spin of the field.

[h] sometimes also called "Millennium" being written actually a year before the (correct) new millennium

[i] The requirement that we have orbifold fixed points is not really necessary for our analysis, which is much more general

Firstly we consider the branes having no matter on them in order to find a suitable vacuum solution. The action of this setup is:

$$S = \int d^4x \int_{-y_3}^{y_3} dy \sqrt{-G}\{-\Lambda + 2M^3 R\} - \sum_i \int_{y=y_i} d^4x V_i \sqrt{-\hat{G}^{(i)}} \tag{10}$$

where $\hat{G}^{(i)}_{\mu\nu}$ is the induced metric on the branes and V_i their tensions. Here we have included negative y and we look for solutions invariant with respect to Z_2 symmetry $y \to -y$.

At this point we demand that our metric respects 4D Poincaré invariance. The metric ansatz with this property is the following: $ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$ Here the "warp" function $\sigma(y)$ is essentially a conformal factor that rescales the 4D component of the metric. It satisfies the following differential equations: $(\sigma')^2 = k^2, \quad \sigma'' = \sum_i \frac{V_i}{12M^3}\delta(y - L_i)$ where $k = \sqrt{\frac{-\Lambda}{24M^3}}$ is a measure of the curvature of the bulk. The brane tensions are tuned to $V_0 = -\Lambda/k > 0$, $V_1 = V_2 = \Lambda/2k < 0$, $V_3 = -\Lambda/k > 0$. It is convenient to introduce 3 dimensionless parameters $x_L = kl_L, \quad x_R = kl_R, \quad x_- = kl_{--}$

We consider now the spectrum that follows from dimensional reduction. This requires we find the spectrum of (linear) fluctuations of the metric:

$$ds^2 = \left[e^{-2\sigma(y)}\eta_{\mu\nu} + \frac{2}{M^{3/2}}h_{\mu\nu}(x,y)\right] dx^\mu dx^\nu + dy^2 \tag{11}$$

We expand the field $h_{\mu\nu}(x,y)$ in terms of the graviton and KK plane wave states : $h_{\mu\nu}(x,y) = \sum_{n=0}^{\infty} h^{(n)}_{\mu\nu}(x)\Psi^{(n)}(y)$ where $\left(\partial_\kappa\partial^\kappa - m_n^2\right) h^{(n)}_{\mu\nu} = 0$ and the gauge is fixed to satisfy $\partial^\alpha h^{(n)}_{\alpha\beta} = h^{(n)\alpha}_{\ \ \alpha} = 0$. The function $\Psi^{(n)}(y)$ obeys a second order differential equation which, after a change of variables, reduces to an ordinary Schrödinger equation:

$$\left\{-\frac{1}{2}\partial_y^2 + V(y)\right\}\hat{\Psi}^{(n)}(y) = \frac{m_n^2}{2}\hat{\Psi}^{(n)}(y), \qquad \hat{\Psi}^{(n)}(y) \equiv \Psi^{(n)}(y)e^{\sigma/2} \tag{12}$$

where the potential $V(y)$ is determined by $\sigma(y)$. Qualitatively it is made up of δ-function potentials (attractive for $''+''$ and repulsive for $''-''$ branes) of different weight depending on the brane tension plus a smoothing term (due to the AdS geometry) that gives the attractive potentials a "volcano" form.

An interesting characteristic of this potential is that it always gives rise to a (massless) zero mode which reflects the fact that Lorenz invariance is preserved in 4D spacetime. This mode is normalizable for finite x_- and becomes non-normalizable in the GSR limit of infinite x_-. The interaction of the linearized gravitons with matter localized on a brane located at some y is given by $\mathcal{L}_{int} = \frac{f(y)}{M^{3/2}}\sum_{n\geq 0}\Psi^{(n)}(y)h^{(n)}_{\mu\nu}(x)T_{\mu\nu}(x)$ with $T_{\mu\nu}$ the energy momentum tensor of the SM Lagrangian and $f(y)$ some universal function. From this expression the Newton potential on a brane is given by

$$U(r) \sim \sum_{n\geq 0}\frac{(\Psi^{(n)})^2(y)}{M^3}\frac{e^{-mr}}{r} \sim \int dm \frac{\Psi_m^2(y)}{M^3}\frac{e^{-mr}}{r} \sim \int dm \rho(m)\frac{e^{-mr}}{r} \tag{13}$$

where the spectral density $\rho(m)$ is determined by the values of normalized wave functions Ψ_m. It is discrete for $x_- = 0$ and any finite x_- and becomes continuous in the GRS limit of infinite x_-.In the case x=0 the ultra-light mass equals to $m_1 = 2ke^{-x_L - x_R}$ which in symmetric case $x_L = x_R$ gives $m_1 = 2ke^{-2x}$. In this case the two wave functions on the + branes are equal, $\Psi_0^2(0) = \Psi_1^2(0)$, to a high accuracy. The masses of the other KK states are found to depend in a different way on the parameter x. The mass of the second state and the spacing Δm between the subsequent states have the form: $m_2 \approx ke^{-x}, \quad \Delta m \approx \varepsilon ke^{-x}$ where ε is a number between 1

and 2. One can see that we just got our Giant see-saw! For length scales less than m_1^{-1}, gravity is generated by the exchange of *both* the massless graviton and the first KK mode, giving the gravitational potential

$$U(r) = C\frac{(\Psi^{(0)})^2(0)}{M^3}\left(\frac{1}{r} + \frac{(\Psi^{(1)})^2(0)}{(\Psi^{(0)})^2(0)}\frac{e^{-m_1 r}}{r}\right) + O(e^{-m_2 r}) \approx 2C\frac{(\Psi^0)^2(0)}{M^3}\frac{1}{r} \tag{14}$$

where C is some constant. We see that the gravitational constant is $G_N = 2C\frac{(\Psi^0)^2(0)}{M^3}$. According to this picture deviations from Newton's law will appear in the submillimeter regime $m_2 r < 1$ as the Yukawa corrections of the second and higher KK states become important. Also the presence of the ultra-light first KK state will give deviations from Newton's law as we probe cosmological scales $m_1 r > 1$ (of the order of the observable universe) with $G_N/2$ instead of G_N. The phenomenological signature of this scenario is that gravitational interactions will appear to become weaker for distances larger than $1/m_1$. Of course when we have asymmetric case the absolute values of the wave functions on positive branes are not equal and one may have arbitrary mixture of massless and ultra-light gravitons contribution to the gravitational constant.

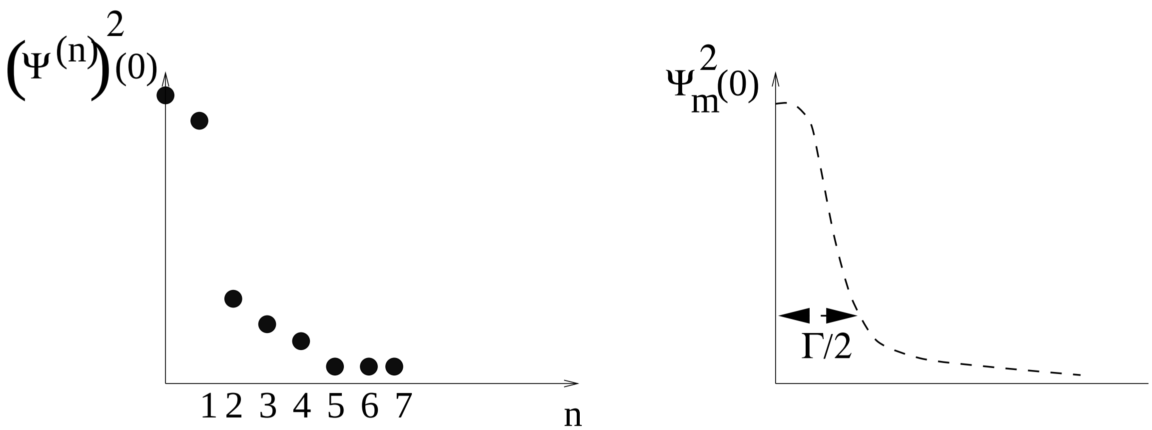

Figure 2: Behaviour $\Psi^2_m(0)$ in a $+-+$ model (discrete spectrum). For a general $+--+$ model the discrete spectrum density increases with the increase of x_- and spectrum becomes continuous in GRS limit.

The structure in the GRS construction is totally different as they do not have normalized modes, but rather a continuous spectrum. In this case there is a "resonance" effect [36] which is due to the fact that the negative brane creates a tunneling factor (the negative brane acts as a repulsive potential) which effectively leads to a resonance in a wave function $\Psi_m(0)$ describing the gravitons $\Psi^2_m(0) = \frac{c}{m^2+\Gamma^2/4} + O(m^4)$ It can be shown that for $r > 1/\Gamma$ the potential is $1/r^2$ and gravity is modified at large distances. But instead of our Giant see-saw we have something else. Let us remind that in multigravity scenario there are three scales: the Planck mass $M_P = r_P^{-1}$, the ultra-light mass scale m_1 (in GRS limit it is the width of resonance) which determines the multigravity crossover scale $R_C = m_1^{-1}$ (cosmological scale !) and the next mass scale m_2 which defines the scale at which gravity is modified at short distances $L = m_2^{-1}$. In GRS limit the relation between them is

$$M_P^2 m_1 = m_2^3, \quad R_C = L\left(\frac{L}{r_P}\right)^2 \tag{15}$$

which is different from Eq. 6. If now we shall take L the same as before the crossover radius R_C will be much bigger than horizon. To get multigravity we must have m_2 in a MeV range. Generic $+--+$ model gives us the following relation: $M_P^{1+\gamma} m_1 = m_2^{2+\gamma}$ with index γ between 0 in $+-+$ limit and 1 in GRS limit.

3 Multigravity Zoo

The idea of multi-localization emerges when one considers configuration of branes such that the corresponding potential $V(z)$ has at least two (δ-function)[j] potential wells . The presence of two wells that each of them could support a bound state has interesting implications. If we consider the above potential wells separated by an infinite distance, then the zero modes are be degenerate i.e. they have the same mass. However if the distance between them becomes finite, then as in quantum mechanics, due to tunneling the degeneracy will be removed and a exponentially small mass splitting will appear between these states. The rest of levels, which are not bound states, although their mass is also modified, do not exhibit the above exponential splitting. The above becomes even more clear if one examine the form of the wave-functions. In the finite distance configuration the wave-function of the zero mode will be the symmetric combination ($\hat{f}_0 = \frac{\hat{f}_0^1+\hat{f}_0^2}{\sqrt{2}}$) of the wave-functions of the zero modes of the two wells (where are infinitely apart) whereas the wave-function of the first KK state will be the antisymmetric combination ($\hat{f}_0 = \frac{\hat{f}_0^1-\hat{f}_0^2}{\sqrt{2}}$). One can see that the absolute value of these wave-functions is almost identical in most of the extra dimension with exception the central region where the antisymmetric passes through zero thought the symmetric has suppressed but non-zero value. Exactly the above is translated to the exponentially small mass difference of these states.

The phenomenon of multi-localization is of particular interest since starting from a problem with only one mass scale we are able to create a second exponentially smaller scale. Obviously the generation of this hierarchy is a key to have ultra-light massive gravitons, i.e. to have multigravity [31,32,33,34,35].

3.1 Universality Classes of Multigravity

Recently another other interesting class of models with massive gravitons was studied in [37,38] (we shall call it DGP model) in which they consider a flat bulk but took into account an effective action induced on a brane in a one-loop approximation. As been suggested long time ago by Sakharov [39] (see also Zeldovich [40]) D-dimensional quantum matter induces a a D-dimensional curvature term (for a review on induced gravity see [41]). An effective action is the sum of four and five dimensional Einstein-Hilbert actions. Phenomenologically this model has the same type of behaviour as GRS - crossover from $1/r$ to $1/r^2$ potential. Actually this is not surprising because gravitational low energy effective Lagrangian for GRS model is precisely a two-term action of DGP model. The GRS model in a long wave-length limit looks like a sandwich with positive brane screened by two negative ones. Effectively this thin slice of AdS space contributes an extra term to an effective action which plays the same role as induced four-dimensional Einstein-Hilbert action - in result one reproduces precisely DGP effective action. However there is one open question - in GRS model there is a radion field (which we shall discuss in the next section) which is a ghost and at first sight there is no ghost fields in DGP model. in GRS model the radion field emerges due to fluctuations of $-1/2$ branes - and of course in a low energy limit these fluctuations are hidden. However it is not completely clear that DGP model does not have radion. First of all there will be induced R^2 terms and it is well known fact that these terms can lead to ghost-like contributions in a graviton propagator. Second problem is related to the sign of the induced cosmological constant which must be cancelled by a brane tension. If the induced cosmological constant is positive one must start from the negative brane and it is not clear if there will be no negative norm states. However it may happen that GRS and DGP models are not equivalent at all. The we have an interesting situation: gravitational sectors are identical but one model has an extra degree of freedom.

[j] In the infinitely thin brane limit that we consider, the wells associated with positive branes are δ-functions

Here we come across a very important concept - *Universality Classes for multigravity.* The concept of universality class is well known and it means that theories in the same universality class may be different at short scales but are the same in the infrared limit. For example second order phase transitions of different physical systems are described by different universality classes which are completely determined by the spectrum of anomalous dimensions and symmetries. It is an interesting question what are universality classes for multigravity, for example can one have two (or more) models with identical low-energy gravitational actions but different with radion/dilaton low-energy actions ? If this is impossible GRS and DGP must be in the same universality class (and for large scale phenomenology it does not matter that there may be two completely different short-distance physics).

Another universality class was discussed in [42] where multigravity in six dimensions was considered. In that case first time one got bounces with flat positive tension branes. Some other models (with AdS branes) we shall mention later, but due to lack of space it is impossible to give any description of these models here.

Let us note that because multigravity is intimately connected with the multilocalization of gravity in multibrane constructions [k] universality classes for multigravity have a geometrical interpretation.

How universality classes can be defined ? A full answer on this question is still unknown but it is clear that the index γ which relates three scales $M_P^{1+\gamma} m_1 = m_2^{2+\gamma}$ is a part of a definition. For example $+-+$ and GRS models are in a different universality classes because they have different $\gamma = 0$ or $\gamma = 1$.

3.2 Can we have massive gravitons ?

Itis well known that massive gravitons have extra polarization states which do not decouple in the massless limit - the so-called van Dam - Veltman - Zakharov discontinuity[45,46]. This property can make multigravity phenomenologically unacceptable as was suggested in [47]. However, an equally generic characteristic of some of these models (but not all of them !) is that they contain moving branes of negative tension. In certain models the radion can help to recover 4D gravity on the brane at intermediate distances. Indeed, the role of the radion associated with the negative tension brane is precisely to cancel the unwanted massive graviton polarizations and recover the correct tensorial structure of the four dimensional graviton propagator [49,48], something also seen from the bent brane calculations of [50,51]. This happens because the radion in this case is a physical ghost because it is has a wrong sign kinetic term. This fact of course makes the construction problematic because the system is probably quantum mechanically unstable. Classically, the origin of the problem is the fact that the weaker energy condition is violated in the presence of moving negative tension branes [52,53].

A way out of this difficulty is to abandon the requirement of flatness of the branes and consider curved ones. A particular example was provided in the $''++''$ model of [54] where no negative tension brane was needed to get multigravity. Moreover, due to the fact that the branes where AdS_4 one could circumvent at least at tree level the van Dam - Veltman - Zakharov theorem [55,56] and the extra polarizations of the massive gravitons where practically decoupled. Of course one can ask the question about the resurrection of these extra polarizations in quantum loops. One loop effects in the massive graviton propagator in AdS_4 were discussed in [57,58]. Of course, purely four-dimensional theory with massive graviton is not well-defined and it is certainly true that if the mass term is added by hand in purely four-dimensional theory a lot of problems will emerge as it was shown in the classical paper of [28]. If however the underlying theory is a higher dimensional one, the graviton(s) mass terms appear dynamically and this is a different story. All quantum corrections must be calculated in a higher-dimensional theory,

[k] multilocalization is a property which can be (with appropriate mass terms) common for all spins [43,44]

where a larger number of graviton degrees of freedom is present naturally (a massless five-dimensional graviton has the same number of degrees of freedom as a massive four-dimensional one).

Moreover, the smoothness of the limit $m \to 0$ is not only a property of the AdS_4 space but holds for any background where the characteristic curvature invariants are non-zero [59,60]. For physical processes taking place in some region a curved space with a characteristic average curvature, the effect of graviton mass is controlled by positive powers of the ratios m^2/R^2 where R^2 is a characteristic curvature invariant (made from Riemann and Ricci tensors or scalar curvature). A very interesting argument supporting the conjecture that there is a smooth limit for phenomenologically observable amplitudes in brane gravity with ultra-light gravitons is based on a very interesting paper [61]

In that paper it was shown that there is a smooth limit for a metric around a spherically symmetric source with a mass M in a theory with massive graviton with mass m for small (*i.e.* smaller than $m^{-1}(mM/M_P^2)^{1/5}$) distances. The discontinuity reveals itself at large distances. The non-perturbative solution discussed in [61] was found in a limited range of distance from the center and it is still unclear if it can be smoothly continued to spatial infinity (this problem was stressed in [28]). Existence of this smooth continuation depends on the full nonlinear structure of the theory. If one adds a mass term by hand the smooth asymptotic at infinity may not exit. However, it seems plausible that in all cases when modification of gravity at large distances comes from consistent higher-dimensional models, the global smooth solution can exist because in this case there is a unique non-linear structure related to the mass term which is dictated by the underlying higher-dimensional theory. In a paper [62] an example of a 5d cosmological solution was discussed which contains an explicit interpolation between perturbative and non-perturbative regimes: a direct analog of large and small distances in the Schwarschild case.

An interesting feature of the above $'' + +''$ model, which only has a dilaton, is that the dilaton survives in the decompactification limit when one of the two branes is sent to infinity [63,64]. This limit was discussed in [65,66,67] and it was found that indeed there is a massive scalar mode in the gravity perturbation spectrum [65]. Although it seems strange to have a dilaton in an infinite extra dimensional model, it is clear that this mode is precisely the remnant of the decompactification process of the compact $'' + +''$ model. This happens as we will show also to multibrane models with flat branes and is related to the fact that the radion has opposite localization properties compared to the ones of the graviton. For more details about radion dynamics of multibrane configurations see [68]

The above multibrane constructions in order to be physically acceptable should incorporate a mechanism which will stabilize the moduli (dilaton, radions) when they have positive kinetic energy and will give them some phenomenologically acceptable mass. This can be achieved by considering for example a bulk scalar field [69,70,71,72] with non trivial bulk potential (for the effect of the Casimir force between the branes see [73]). A general condition that guarantees stabilization of the dilaton in the case of maximally symmetric branes was derived in [63] and restricts the sum of the effective tensions of the branes and the leftover curvature of the brane. The moduli stabilization has greater importance in the context of brane cosmology where it was found that it played a crucial role in deriving normal cosmological evolution on the branes [74,75,76]. A non-perturbative analysis of the dilaton two brane models can be found in [77].

When the radion has negative kinetic energy is still not clear whether one can speak about stabilization of these systems. They are probably unstable at the quantum level and no one has attempted to estimate their life-time. Actually it is not completely clear if unstable negative norm states are really forbidden - the problem is that if there are several excitations in the same channel and they all have finite width an interesting thing may happen - the distributions with negative norms will sit inside positive norm resonances. In other words total spectral density may be still positive - unstable negative norm states will simply reduce total spectral

density. Perhaps one can think that unwanted polarizations of massive gravitons in physical amplitudes will be cancelled by negative norm states which becomes resonances and in result one will have totally positive spectral density. This is still an open question which deserves a further investigation.

4 Observable effects

Observation of modifications of gravity at ultra-large scale could be a striking signal of such a possibility. There are several papers in which experimental tests were discussed. In [78] CMB measurements were discussed which are sensitive to a large scale limit of gravitational interactions. Lensing at cosmological scales due to multigravity was discussed in [79] and SN1A data and CMB for multigravity was considered in [80]. A lot of astrophysical constraints on modifying gravity at large distances was discussed in [27]. One of the reason why multigravity can modify CMB is that it leads to a large distance modifications of the curvature. When traced back at the time of inflation this gives rise to a dispersed frequency for the cosmic perturbations. The perturbation field is minimally coupled to gravity and it long wavelength modes are influenced by the modifications in the background curvature. The analysis of the CMB spectrum for the whole range of modes [80] reveals that the spectrum deviates from scale-invariance and is extremely sensitive to large distance physics (because long wavelength modes dominate the spectrum) and the choice of the initial conditions, but does not depend in the details of short-distance physics (transplanckian modes).This deviation is small for curvature modifications around the last scattering horizon scale.

There are other experiments to study multigravity, for example precision tests, but here we do not have time to discuss it.

4.1 Multigravity and Cosmology:Dark Matter and Quintessence

One of very striking features of multigravity is that it gives us a some sort of a dark matter. The origin of this dark matter is very straightforward - this is just matter from other branes! So it is by definition "dark" if we assume that all SM fields propagates along the same brane. But why shall we see matter from other branes ? Let's concentrate on 5-dimensional case. We have multilocalization and the massless graviton is localized on several branes - which means that it interaction with matter is of the form:

$$h^0_{\mu\nu}(x) \sum_i \Psi_0(y_i) T^i_{\mu\nu}(x) \tag{16}$$

where $T^i_{\mu\nu}(x)$ are stress-energy tensors of matter localized on different branes located at points y_i and $\Psi_0(y)$ is the massless graviton profile (which must be of the bounce form). The coupling for the next excited state is given by

$$h^1_{\mu\nu}(x) \sum_i \Psi_1(y_i) T^i_{\mu\nu}(x) \tag{17}$$

where $\Psi_1(y)$ is the first ultra-light graviton profile, etc.. Because $\Psi_n(y_i)$ are quite different we see that we have different effective source. Let us study the simplest model with two branes, when on a first brane $\Psi_0(y_1) = \Psi_1(y_1) = 1$ and on a second one $\Psi_0(y_2) = -\Psi_1(y_2) = 1$ the coupling is

$$\left(h^0_{\mu\nu}(x) + h^1_{\mu\nu}(x)\right) T^1_{\mu\nu}(x) + \left(h^0_{\mu\nu}(x) - h^1_{\mu\nu}(x)\right) T^2_{\mu\nu}(x) \tag{18}$$

On a first brane observable metric is $h^0_{\mu\nu}(x) + h^1_{\mu\nu}(x)$ and we see that matter from second brane does not contribute to it. But this is true only for distances smaller than R_C when we can not

see difference between two gravitons. For larger distances the ultra-light graviton effectively decouples and we can only see the massless one. But $h^0_{\mu\nu}(x)$ is produced by the *sum* of stress-energy tensors from both branes $T^1_{\mu\nu}(x) + T^2_{\mu\nu}(x)$. Thus multigravity opens a window in extra dimensions - we start to feel gravitationally matter which is localized on other branes ! Moreover - we can not see this matter at intermediate scales - the coherent combination of massless and light gravitons screen this matter from us. This window opens ONLY at large scale.

This is indeed a dark matter. In an example described above we shall see two times more matter -but at the same time our gravitational constant is smaller by a factor of 2 so overall gravity is not stronger. But we can consider more sophisticated models with more than 2 branes and in principle it is possible to get an amplification of the gravitational force. It remains to be seen if one can construct a model which will be phenomenologically acceptable to explain rotational curves.

But in any case we see that multigravity means dark matter, the question is can it account for ALL dark matter ? This question is closely related to another one: what effect does have multigravity on cosmology ? The Hubble law relates rate of expansion to matter density $(\dot{a}/a)^2 \sim G_N \rho$. Change in G_N leads to change in Hubble law. This means that for scale factors $a(t)$ bigger and smaller the crossover radius R_C we have to use different gravitational constants. Thus crossover in Newton law leads to a crossover in cosmological evolution. Does it mean that multigravity also means Quintessence ? Is multigravity important for cosmological constant problem ?

5 Conclusion

It seems it is a good time to stop here. Answers on all these exciting questions are not known (or known only partially). This is what people are working on right now and hopefully one day we shall know these answers. If they are negative - well, this is just another interesting possibility which has nothing to do with real world (or we have some small admixture of ultralight states, but nothing drastic). But if the answers are positive.....

Acknowledgments

This talk was given at Moriond meeting and memories of this exciting (as usually) event helped a lot during the process of writing. I am grateful for all organizers of this wonderful meeting for very stimulating and relaxed atmosphere. Special thanks are to Claude Barthlemy - who was persistent enough to persuade me to write this text after two deadlines and patient enough for me not to feel guilty. I am indebted to Stavros Mouslopoulos, Antonios Papazoglou and Graham G. Ross in collaboration with whom most of the results presented here were obtained. I would like to thank Thibault Damour, Panagiota Kanti, Laura Mersini, Keith Olive, Luigi Pilo, Bayram Tekin and Arkady Vainshtein for very stimulating discussions. This work is supported in part by the PPARC rolling grant PPA/G/O/1998/00567, by the EC TMR grants HRRN-CT-2000-00148 and HPRN-CT-2000-00152.

References

1. T. Regge and C. Teitelboim, *In *Trieste 1975, Proceedings, Marcel Grossmann Meeting On General Relativity*, Oxford 1977, 77-87.*
2. K. Akama, Lect. Notes Phys. **176**, 267 (1982) [hep-th/0001113].
3. V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, 136 (1983).
4. M. Visser, Phys. Lett. B **159**, 22 (1985) [hep-th/9910093].
5. E. J. Squires, Phys. Lett. B **167**, 286 (1986).

6. G. W. Gibbons and D. L. Wiltshire, Nucl. Phys. B **287** (1987) 717.
7. R. Cordero and A. Vilenkin, hep-th/0107175.
8. A. Davidson, Class. Quant. Grav. **16** (1999) 653 [gr-qc/9710005].
9. S. Deser, F. A. Pirani and D. C. Robinson, Phys. Rev. D **14** (1976) 3301.
10. A. Friedman, Rev. Mod. Phys. **37** (1965) 201; J. Rosen, *ibid* 204; R. Penrose, *ibid* 215; D.W. Joseph, *ibid* 225; Y. Ne'eman *ibid* 227.
11. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B **429**, 263 (1998) [hep-ph/9803315].
12. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B **436**, 257 (1998) [hep-ph/9804398].
13. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. D **59**, 086004 (1999) [hep-ph/9807344].
14. P. Horava and E. Witten, Nucl. Phys. B **460**, 506 (1996) [hep-th/9510209].
15. E. Witten, Nucl. Phys. B **471**, 135 (1996) [hep-th/9602070].
16. P. Horava and E. Witten, Nucl. Phys. B **475**, 94 (1996) [hep-th/9603142].
17. A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, Phys. Rev. D **59**, 086001 (1999) [hep-th/9803235].
18. A. Lukas, B. A. Ovrut and D. Waldram, Phys. Rev. D **60**, 086001 (1999) [hep-th/9806022].
19. M. Gogberashvili, Mod. Phys. Lett. A **14**, 2025 (1999) [hep-ph/9904383].
20. L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [hep-th/9906064].
21. L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221].
22. V. A. Rubakov, hep-ph/0104152.
23. S. Hildebrandt and A. Tromba, *The parsimonious universe: shape and form in the natural world*, Copernicus, Springer-Verlag New York, Inc., 1996
24. B. Tekin, hep-th/0106134.
25. M. Milgrom, astro-ph/9810302.
26. M. Milgrom, astro-ph/9801301.
27. A. Aguirre, C. P. Burgess, A. Friedland and D. Nolte, hep-ph/0105083.
28. D. G. Boulware and S. Deser, Phys. Rev. D **6**, 3368 (1972).
29. A. S. Goldhaber and M. M. Nieto, Phys. Rev. D **9**, 1119 (1974).
30. S. M. Carroll, Living Rev. Rel. **4** (2001) 1 [astro-ph/0004075].
31. I. I. Kogan, S. Mouslopoulos, A. Papazoglou, G. G. Ross and J. Santiago, Nucl. Phys. B **584**, 313 (2000) [hep-ph/9912552].
32. R. Gregory, V. A. Rubakov and S. M. Sibiryakov, Phys. Rev. Lett. **84**, 5928 (2000) [hep-th/0002072].
33. I. I. Kogan and G. G. Ross, Phys. Lett. B **485**, 255 (2000) [hep-th/0003074].
34. S. Mouslopoulos and A. Papazoglou, JHEP **0011**, 018 (2000) [hep-ph/0003207].
35. I. I. Kogan, S. Mouslopoulos, A. Papazoglou and G. G. Ross, Nucl. Phys. B **595**, 225 (2001) [hep-th/0006030].
36. C. Csaki, J. Erlich and T. J. Hollowood, Phys. Rev. Lett. **84** (2000) 5932 [hep-th/0002161].
37. G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [hep-th/0005016].
38. G. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001) [hep-th/0008054].
39. A. D. Sakharov, Sov. Phys. Dokl. **12** (1968) 1040 [Dokl. Akad. Nauk Ser. Fiz. **177** (1968) 70].
40. Y. B. Zeldovich, JETP Lett. **6** (1967) 316 [Pisma Zh. Eksp. Teor. Fiz. **6** (1967) 883].
41. S. L. Adler, Rev. Mod. Phys. **54** (1982) 729 [Erratum-ibid. **55** (1982) 837].
42. I. I. Kogan, S. Mouslopoulos, A. Papazoglou and G. G. Ross, hep-th/0107086.
43. S. Mouslopoulos, JHEP **0105** (2001) 038 [hep-th/0103184].
44. I. I. Kogan, S. Mouslopoulos, A. Papazoglou and G. G. Ross, hep-ph/0107307.
45. H. van Dam and M. Veltman, Nucl. Phys. B **22**, 397 (1970).

46. V.I. Zakharov, JETP Lett. **12** (1970) 312.
47. G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **484**, 112 (2000) [hep-th/0002190].
48. L. Pilo, R. Rattazzi and A. Zaffaroni, JHEP **0007**, 056 (2000) [hep-th/0004028].
49. G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **484**, 129 (2000) [hep-th/0003054].
50. C. Csaki, J. Erlich and T. J. Hollowood, Phys. Lett. B **481**, 107 (2000) [hep-th/0003020].
51. R. Gregory, V. A. Rubakov and S. M. Sibiryakov, Phys. Lett. B **489**, 203 (2000) [hep-th/0003045].
52. D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, Adv. Theor. Math. Phys. **3**, 363 (1999) [hep-th/9904017].
53. E. Witten, hep-ph/0002297.
54. I. I. Kogan, S. Mouslopoulos and A. Papazoglou, Phys. Lett. B **501**, 140 (2001) [hep-th/0011141].
55. I. I. Kogan, S. Mouslopoulos and A. Papazoglou, Phys. Lett. B **503**, 173 (2001) [hep-th/0011138].
56. M. Porrati, Phys. Lett. B **498**, 92 (2001) [hep-th/0011152].
57. F. A. Dilkes, M. J. Duff, J. T. Liu and H. Sati, hep-th/0102093.
58. M. J. Duff, J. T. Liu and H. Sati, hep-th/0105008.
59. T. Damour and I. I. Kogan, work in progress.
60. I. I. Kogan, S. Mouslopoulos and A. Papazoglou, work in progress.
61. A. I. Vainshtein, Phys. Lett. **39B**, 393 (1972).
62. C. Deffayet, G. Dvali, G. Gabadadze and A. Vainshtein, hep-th/0106001.
63. A. Papazoglou, Phys. Lett. B **505**, 231 (2001) [hep-th/0102015].
64. Z. Chacko and P. J. Fox, hep-th/0102023.
65. A. Karch and L. Randall, Int. J. Mod. Phys. A **16**, 780 (2001) [hep-th/0011156].
66. A. Miemiec, hep-th/0011160.
67. M. D. Schwartz, Phys. Lett. B **502**, 223 (2001) [hep-th/0011177].
68. I. I. Kogan, S. Mouslopoulos, A. Papazoglou and L. Pilo, hep-th/0105255.
69. W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [hep-ph/9907447].
70. O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D **62**, 046008 (2000) [hep-th/9909134].
71. P. Kanti, K. A. Olive and M. Pospelov, Phys. Lett. B **481**, 386 (2000) [hep-ph/0002229].
72. C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. D **63**, 065002 (2001) [hep-th/0008151].
73. R. Hofmann, P. Kanti and M. Pospelov, hep-ph/0012213.
74. P. Kanti, I. I. Kogan, K. A. Olive and M. Pospelov, Phys. Lett. B **468**, 31 (1999) [hep-ph/9909481].
75. C. Csaki, M. Graesser, L. Randall and J. Terning, Phys. Rev. D **62**, 045015 (2000) [hep-ph/9911406].
76. P. Kanti, I. I. Kogan, K. A. Olive and M. Pospelov, Phys. Rev. D **61**, 106004 (2000) [hep-ph/9912266].
77. P. Binetruy, C. Deffayet and D. Langlois, hep-th/0101234.
78. P. Binetruy and J. Silk, Phys. Rev. Lett. **87** (2001) 031102 [astro-ph/0007452].
79. J. Uzan and F. Bernardeau, hep-ph/0012011.
80. M. Bastero-Gil and L. Mersini, astro-ph/0107256.